Illustrates the Bayesian decision-tree approach as useful tool in making auditing decisions.

Making Auditing Decisions:

The Bayesian Decision-tree Approach

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This article attempts to illustrate the use of the Bayesian decision-tree approach in the analysis and evaluation of decisions related to the audit process. The suggestion to use Bayesian statistics in auditing is not new. Various suggestions have been made in different forms and different auditing contexts (e.g. on attribute sampling[1], sample size[2], on variable sampling[3], and on compliance testing[4]). The objective of this article is to build upon the previous literature by offering additional insights into the application of Bayesian statistics in auditing and presenting the Bayesian decision-tree approach in an operational form.

The first section of the article briefly reviews the audit process and identifies some decision problems related to the process. The second section describes the use of the Bayesian decision-tree approach in analysing and evaluating decisions. Finally, the third section illustrates the analysis and evaluation of two auditing decisions (concerning substantive testing and the review of other auditors' work) using decision trees.

The Audit Process and Decision Making

The final outcome of the audit of a company's financial statements is the issuance of an audit report containing "the auditor's opinion on the presentation in the financial statements of the entity's financial position and the results of its operations"[5, para. 8]. The opinion that is expressed by the auditor is based on his/her review and assessment of the audit evidence obtained during the audit. It may be an unqualified opinion, a qualified opinion, an adverse opinion, or a disclaimer of opinion[5, para. 14].

The audit evidence on which the auditor's opinion is based is obtained mainly through the performance of compliance and substantive procedures. Compliance procedures are tests designed to obtain reasonable assurance that those internal controls on which audit reliance is to be placed are in effect. Substantive procedures are designed to obtain evidence as to the completeness, accuracy, and validity of the data produced by the accounting system[6, para. 1]. Generally, if the results of the compliance tests are favourable (i.e. indicating that internal controls can be relied upon), then less extensive substantive tests are needed.

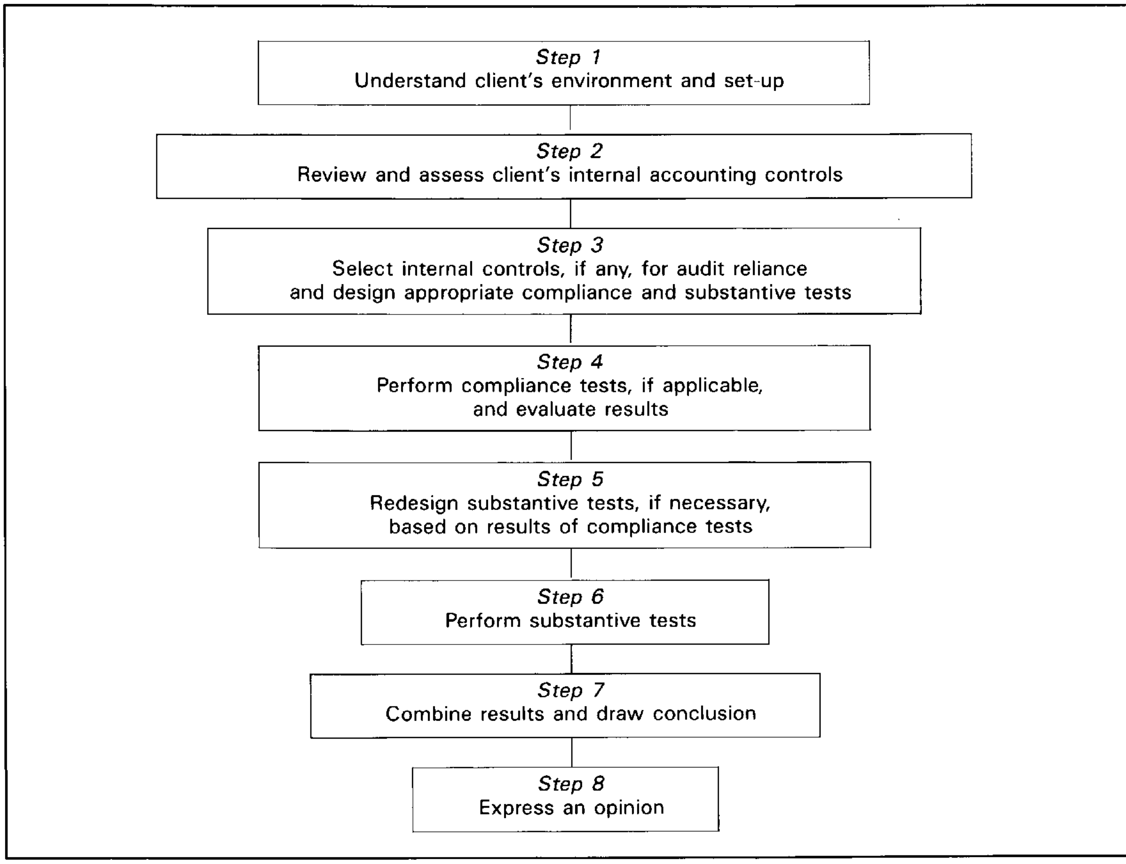
The process of accumulating audit evidence, reviewing and assessing the evidence, and arriving at an opinion on the client's financial statements is often referred to as the audit process. An overview of this audit process is shown in Figure 1.

As can be seen from Figure 1, before the auditor is in a position to design appropriate compliance and substantive tests to obtain audit evidence (step 3) he/she should first seek to have a reasonable understanding of the client's environment (such as its economic and industrial setting) and its set-up (such as its operations, organizational structure, and accounting system). In addition, a review of the client's system of internal accounting controls and an assessment of the extent to which it can be relied upon (step 2) is needed before the auditor can select the internal controls, if any, for compliance testing.

After carrying out the compliance tests (if any) and evaluating the results (step 4), the auditor may need to redesign the substantive tests (step 5) before conducting them (step 6) if the planned reliance on the internal controls is not supported by the compliance test results. After conducting the substantive tests, all the results (i.e. all audit evidence obtained through compliance tests, if any, and substantive tests) are combined to arrive at a conclusion (step 7). Then based on this conclusion, an opinion on the client's financial statements is expressed (step 8).

The ultimate decision made by the auditor is his/her opinion at the end of the audit process of whether the client's financial statements present a true and fair view of its operations and financial condition. However, besides this ultimate decision, there are other decisions that the

Figure 1. *An Overview of the Audit Process*



auditor has to make during the audit process itself. For example, after reviewing and assessing the client's system of internal controls, the auditor has to decide on the extent of reliance, if any, to be placed on such controls. In the design of appropriate compliance and substantive procedures, the auditor has to decide on the type of tests to perform, and for each test, the sample size to select, the particular items to include in the sample, and the timing of the test.

After carrying out the compliance tests on internal controls, the auditor has to decide whether the test results substantiate the planned audit reliance on these controls. If the planned audit reliance is not justified, then he/she has to decide on the extent to which related substantive tests should be increased. Also, after carrying out the substantive tests, the auditor has to decide whether all the audit evidence obtained so far is sufficient for arriving at a conclusion concerning the client's financial statements. If it is not sufficient, then he/she has to decide on other tests to obtain more evidence.

Many of these auditing decisions are quite problematic because of factors outside the auditor's control. In addition some of these auditing decisions, if not all, are dependent on other auditing decisions. For example, the decision on the types of compliance and substantive tests to be carried out depends on the auditor's decision concerning the extent to which the company's internal controls can be relied upon. If the auditor decides that the company's internal controls are not reliable then extensive compliance tests will serve little purpose. In such a situation, the auditor would perform more extensive substantive tests to obtain the required audit evidence on which to base his/her opinion. On the other hand, if the auditor decides that the company's internal controls are highly reliable, he/she would choose to carry out extensive compliance tests in the hope that the test results would corroborate his/her decision and thus justify a reduction in the substantive tests.

In making such decisions concerning compliance and substantive tests, the costs of performing the various tests would be an important factor[7]. For example, if the auditor believes that the company's internal controls can be relied upon and (1) the cost of a set of substantive procedures which reflect no reliance on internal controls is higher than (2) the cost of a set of extensive compliance procedures and reduced substantive procedures based on the successful tests of compliance of internal controls, he/she would opt for the latter, (i.e. [2]). However, since there is no guarantee that the tests of compliance would be successful, the auditor would also take into consideration (besides the cost factor) the chance or probability that the compliance tests would be successful. If the compliance tests are unsuccessful, the auditor would have incurred additional cost in performing the extensive compliance tests without obtaining the benefit of cost savings arising from the reduction in substantive tests.

In other words, the decision concerning compliance and substantive tests is a decision problem where the outcome (i.e. the costs of the tests) is not only dependent on the action (i.e. to perform or not to perform extensive compliance tests) taken by the auditor but also on an event outside the auditor's control (i.e. the results of the compliance tests). Similarly, most of the other decision problems encountered by the auditor are also of this nature, where the choice of a particular alternative course of action is influenced by the chance or probabilities for the occurrences of various possible uncontrollable events. This will be discussed in greater detail later.

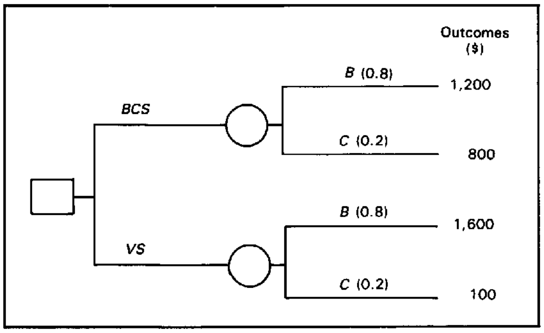
A decision making tool which is very useful for analysing and evaluating decisions of this nature is the decision tree. The following section illustrates the use of decision trees in analysing and evaluating such decisions.

Using Decision Trees to Analyse Decisions

A decision tree is a pictorial representation of a decision problem. To illustrate the use of decision trees in analysing decision problems, consider a simple decision of whether to invest $1,000 in blue-chip stock or in venture stock. For simplicity, assume that there are only two possible stock-market trends: boom or collapse. If there is a boom in the stock market, the pay-offs for investing in the blue-chip stock and venture stock are $1,200 and $1,600 respectively. If the stock market collapses, the pay-offs for investing in the blue-chip stock and venture stock are $800 and $100 respectively. Furthermore, assume that there is a 0.2 chance that the stock market will collapse. This decision problem can be represented by the decision tree shown in Figure 2.

The square in Figure 2, known as a decision node, represents a point in time when the decision maker has to choose one alternative from a finite number of

Figure 2. *Using a Decision Tree to Depict a Decision Problem*



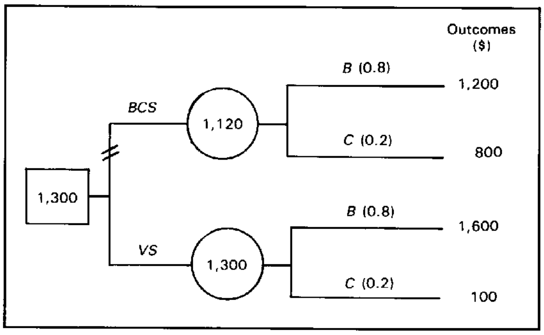
alternative courses of action. In this case, the alternative courses of action are to invest in Blue-Chip Stock (BCS) or to invest in Venture Stock (VS), as represented by the two branches emanating from the decision node. The circle, known as the chance node, represents a chance point, i.e. a point in time when one out of a finite number of non-controllable events or states of nature will occur. In this case, regardless of whether the decision is to invest in blue-chip stock or to invest in venture stock, one out of two possible uncontrollable events will occur, a stock-market boom (B) or a stock-market collapse (C). These uncontrollable events are represented by branches emanating from the chance nodes with the chance or probability of their occurrences (i.e. Pr(B) = 0.8 and Pr(C) = 0.2) indicated within parentheses. The outcomes indicated at the end of each branch are the pay-offs resulting from the choice of a particular alternative course of action and the occurrence of a particular uncontrollable event. In this case, if the choice is to invest in blue-chip stock, the pay-offs are $1,200 if there is a stock-market boom and $800 if there is a stock-market collapse. If the choice is to invest in venture stock, the pay-offs are $1,600 if there is a stock-market boom and $100 if there is a stock-market collapse.

Having drawn a decision tree to depict all the relevant elements of the decision problem, the next step is to evaluate the various alternative courses of actions in terms of both pay-offs and probabilities using the concept of expected value (EV). An expected value is a probability-weighted average. The expected values for investing in blue-chip stock EV(BCS) and investing in venture stock EV(VS) can be derived as follows:

$$
\begin{aligned}
EV(BCS) &= Pr(B)*1{,}200 + Pr(C)*800 \\
&= 0.8*1{,}200 + 0.2*800 \\
&= 1{,}120
\end{aligned}
$$

$$
\begin{aligned}
EV(VS) &= Pr(B)*1{,}600 + Pr(C)*100 \\
&= 0.8*1{,}600 + 0.2*100 \\
&= 1{,}300.
\end{aligned}
$$

Figure 3. *Using a Decision Tree to Evaluate a Decision*



Based on the above expected values, the better alternative is to invest in venture stock. Figure 3 shows how this evaluation and final decision are incorporated in the decision tree. The expected values for the two alternatives are entered in their respective chance nodes. To indicate that the alternative of investing in venture stock is a better one, its expected value of $1,300 is entered in the decision node and the branch representing the other alternative of investing in blue-chip stock is crossed out using this symbol //.

Additional Information and Posterior Probabilities

In many decision problems, the decision maker is often given the opportunity to acquire additional information that would permit more accurate assessments of the probabilities for the occurrence of the uncontrollable events. For example, in the above illustration, the decision maker could perhaps obtain the service of a market analyst to forecast what the stock-market trend would be. Since there is always a cost involved in acquiring additional information, the decision maker must evaluate whether the value of such additional information is worth more than the cost of acquiring it. Decision trees can also be used to represent and evaluate such decisions.

Before illustrating the evaluation of such decisions with the aid of decision trees, the concept of prior probabilities and posterior probabilities needs to be introduced. Prior probabilities refer to the initial estimates of the chance or probability for the occurrence of the various alternative uncontrollable events. Posterior probabilities refer to the revised estimates of the chance or probability for the occurrence of the various alternative uncontrollable events in the light of additional information. Using the above illustrative example, the prior probabilities of a boom in the stock market and a collapse in the stock market are the initial estimates of 0.8 and 0.2, respectively, i.e. Pr(B) = 0.8 and Pr(C) = 0.2. The posterior probabilities are the revised estimates of the probability of a stock-market boom and a stock-market collapse if the market analyst forecasts either a stock-market boom (FB) or a stock-market collapse (FC), i.e. Pr(B|FB), Pr(C|FB) and Pr(B|FC), Pr(C|FC).

Posterior probabilities can be derived using Bayes' formula if estimates concerning the reliability of the additional information are available. For example, from past experience it may be found that when there is a stock-market boom, there is a 0.9 chance or probability that the market analyst forecasts a boom, i.e. Pr(FB|B) = 0.9 and Pr(FC|B) = 0.1, and when there is a stock-market collapse, there is a 0.2 chance or probability that the market analyst forecasts a boom, i.e. Pr(FB|C) = 0.2 and Pr(FC|C) = 0.8. Then the posterior probabilities of a stock-market boom can be derived as follows, using Bayes' formula:

$$\begin{aligned} Pr(B|FB) &= [Pr(B)*Pr(FB|B)]/[Pr(B)*Pr(FB|B) + Pr(C)*Pr(FB|C)] \\ &= (0.8*0.9)/(0.8*0.9 + 0.2*0.2) \\ &= 0.947 \end{aligned}$$

$$\begin{aligned} Pr(B|FC) &= [Pr(B)*Pr(FC|B)]/[Pr(B)*Pr(FC|B) + Pr(C)*Pr(FC|C)] \\ &= (0.8*0.1)/(0.8*0.1 + 0.2*0.8) \\ &= 0.333. \end{aligned}$$

Similarly, the posterior probabilities of a stock-market collapse can be derived using Bayes' formula or simply by subtracting the posterior probabilities of a stock-market boom from one as shown below:

$$\begin{aligned} Pr(C|FB) &= 1 - Pr(B|FB) \\ &= 1 - 0.947 \\ &= 0.053 \end{aligned}$$

$$\begin{aligned} Pr(C|FC) &= 1 - Pr(B|FC) \\ &= 1 - 0.333 \\ &= 0.667. \end{aligned}$$

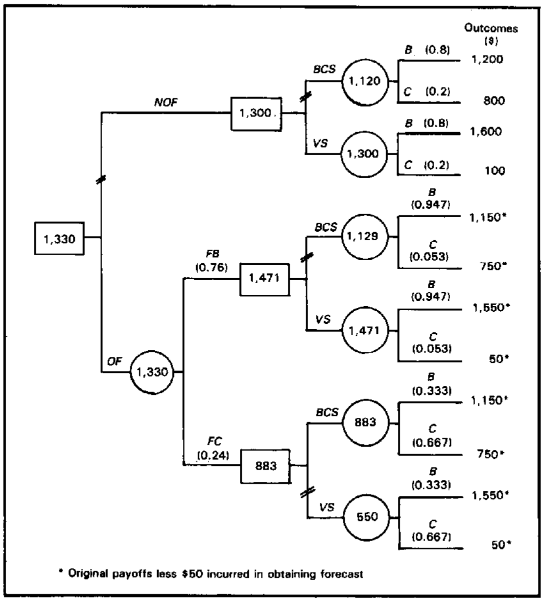
In addition to the posterior probabilities for the occurrence of the uncontrollable events, the probabilities of obtaining the different information signals from the information-seeking exercise must also be derived before an evaluation of the value of the additional information can be made. In the case of the above, the probability Pr(FB), that the market analyst forecasts a stock-market boom, and the probability Pr(FC), that the market analyst forecasts a stock-market collapse, can be derived as follows:

$$\begin{aligned} Pr(FB) &= Pr(B)*Pr(FB|B) + Pr(C)*Pr(FB|C) \\ &= 0.8*0.9 + 0.2*0.2 \\ &= 0.76 \end{aligned}$$

$$\begin{aligned} Pr(FC) &= 1 - Pr(FB) \\ &= 1 - 0.76 \\ &= 0.24. \end{aligned}$$

To illustrate how decisions on whether to obtain additional information can be evaluated using decision trees, a decision tree incorporating the decision on whether to

Figure 4. *Decision Tree Incorporating a Decision on Whether to Obtain a Forecast of Stock Market Trend*



incur a cost, say $50, to obtain a forecast of the stock-market trend from the market analyst is shown in Figure 4, where prior to the decision on whether to invest in blue-chip stock (BCS) or venture stock (VS), a decision on whether to obtain a forecast (OF), or not to obtain a forecast (NOF) of the stock-market trend has to be made. If the decision is not to obtain a forecast, the alternative of investing in venture stock with an expected value of $1,300 is better than the alternative of investing in blue-chip stock with an expected value of $1,120. In other words, the expected value of not obtaining a forecast is $1,300, i.e. EV(NOF) = 1,300.

However, if the decision is to obtain a forecast, then whether to invest in blue-chip stock or venture stock would depend on what is forecast. If a boom in the stock market is forecast (FB), then investing in venture stock with an expected value of $1,471 is the better alternative. However if a collapse in the stock market is forecast (FC), then investing in blue-chip stock with an expected value of $883 is the better alternative. Therefore, the expected value of obtaining a forecast EV(OF) can be computed as follows:

$$\begin{aligned} EV(OF) &= Pr(FB)*1{,}471 + Pr(FC)*883 \\ &= 0.76*1{,}471 + 0.24*883 \\ &= 1{,}330. \end{aligned}$$

In terms of expected values, the alternative of obtaining a forecast is better than the alternative of not obtaining a forecast. So the optimal decisions are to obtain a forecast and to invest in venture stock if the forecast predicts a market boom or to invest in blue-chip stock if the forecast predicts a market collapse. This will result in the higher expected value of $1,330 as compared with the expected value of $1,300 if the decisions are not to obtain a forecast and to invest in venture stock (the next best courses of action).

The decision tree is thus a useful tool for analysing decisions where the outcomes are dependent on both the alternative courses of action that are chosen, as well as on the alternative uncontrollable events that can subsequently occur. The following section illustrates the use of the Bayesian decision-tree approach in analysing and evaluating certain decisions in the audit process; namely, a decision concerning substantive testing and a decision pertaining to the review of other auditors' work.

Example 1. Substantive Testing

Problem

Suppose that an auditor (in the process of performing a financial audit) has to determine if an account balance is fair [F] or not fair [NF]. For this purpose, the auditor performs a substantive test on the account balance to decide whether to accept [A] or not to accept [NA] the balance as fair. From past experience, the prior probability of the account balance being fair is 0.90 and the prior probability of it being not fair is 0.10, i.e. Pr(F) = 0.90 and Pr(NF) = 0.10.

If the account balance is fair and the auditor accepts it as fair or if the account balance is not fair and the auditor does not accept it as fair, then the auditor's decision is correct and the expected cost of such a decision is zero, i.e. $E(C|F,A) = 0$ and $E(C|NF,NA) = 0$. However, if the account balance is not fair and the auditor accepts it as fair, then the consequences of such a decision can be severe (e.g. issuing an incorrect audit opinion). The auditor estimates the expected cost of such an incorrect decision (a β error) to be $100,000, i.e. $E(C|NF,A) = 100{,}000$. On the other hand, if the account balance is fair and the auditor does not accept it as fair, then the decision is incorrect (an α error). But the consequences of an α error (e.g. unnecessary audit work) are not as serious as those of the β error. Accordingly, the auditor estimates the expected cost of such an incorrect decision to be $10,000, i.e. $E(C|F,NA) = 10{,}000$.

Suppose that the auditor can perform two types of substantive tests, namely, a high-level substantative test (HST) and a low-level substantive test (LST), and that the costs of the two types of substantive tests are $15,000 and $5,000 respectively. Suppose further that results from the substantive tests can give two types of indications — a positive indication that the account balance is fair (denoted P) or a non-positive indication that the account balance is not fair (denoted NP). From past experience, the auditor estimates that when the account balance is fair, the probabilities of getting indications P if HST and LST are performed are 0.95 and 0.80 respectively, i.e. Pr(P|F,HST) = 0.95 and Pr(P|F,LST) = 0.80. The corresponding probabilities when the account balance is not fair are 0.10 and 0.25 respectively, i.e. Pr(P|NF, HST) = 0.10 and Pr(P|NF, LST) = 0.25. Furthermore, past experience also shows that HST is performed 60 per cent of the time, regardless of whether the account balance is fair or not, i.e. Pr(HST|F) = 0.60 and Pr(HST|NF) = 0.60.

Given the above information, the auditor wants to decide if he or she should perform a high-level or low-level substantive test. It can be assumed that the auditor accepts the account balance as correct when positive indications are received from substantive tests and does not accept the account balance as correct when non-positive indications are received from substantive tests.

Solution

Given:

Pr(F) = 0.90
Pr(NF) = 0.10
Pr(P|F,HST) = 0.95
Pr(P|NF,HST) = 0.10
Pr(P|F,LST) = 0.80
Pr(P|NF,LST) = 0.25
Pr(HST|F) = 0.60
Pr(HST|NF) = 0.60.

The calculation of the various conditional probabilities required for the decision tree may proceed as follows: Let Φ_i, for $i=1,2$, denote the events F and NF respectively; Ω_j, for $j=1,2$, denote the events P and NP respectively; Σ_k, for $k=1,2$, denote the events HST and LST respectively. The information given above corresponds to $\Pr(\Phi_i)$, $\Pr(\Sigma_k|\Phi_i)$ and $\Pr(\Omega_j|\Phi_i,\Sigma_k)$, from which the required conditional probabilities for the decision tree can be derived as shown below:

Step 1. Compute $\Pr(\Phi_i|\Sigma_k)$ using Bayes' Theorem:

$$\Pr(\Phi_i|\Sigma_k) = \frac{\Pr(\Sigma_k|\Phi_i)\Pr(\Phi_i)}{\sum_{i=1}^{2}\Pr(\Sigma_k|\Phi_i)\Pr(\Phi_i)}.$$

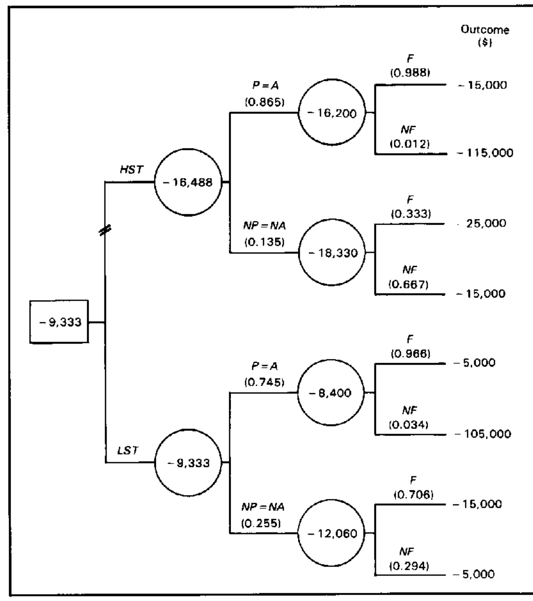
Step 2. Using the results from Step 1,

$$\Pr(\Omega_j|\Sigma_k) = \Pr(\Omega_j|\Phi_1,\Sigma_k)\Pr(\Phi_1|\Sigma_k) + \Pr(\Omega_j|\Phi_2,\Sigma_k)\Pr(\Phi_2|\Sigma_k).$$

Step 3. Using the results from the previous two steps and the information given,

$$\Pr(\Phi_i|\Omega_j,\Sigma_k) = \frac{\Pr(\Omega_j|\Phi_i,\Sigma_k)\Pr(\Phi_i|\Sigma_k)}{\Pr(\Omega_j|\Sigma_k)}.$$

Figure 5. *A Decision Tree to Evaluate the Level of Substantive Test*



Applying these relations, the following required probabilities are obtained (readers interested in the detailed computations can write to the authors for information):

Pr(P|HST)	= 0.865	Pr(NP|HST)	= 0.135
Pr(P|LST)	= 0.745	Pr(NP|LST)	= 0.255
Pr(F|P,HST)	= 0.988	Pr(NF|P,HST)	= 0.012
Pr(F|NP,HST)	= 0.333	Pr(NF|NP,HST)	= 0.667
Pr(F|P,LST)	= 0.966	Pr(NF|P,LST)	= 0.034
Pr(F|NP,LST)	= 0.706	Pr(NF|NP,LST)	= 0.294

Based on the computations above, the decision tree is presented in Figure 5. As can be seen, the conclusion that can be drawn from the decision tree is that it is better to perform a low-level substantive test (LST) with an expected cost of $9,333 than a high-level substantive test (HST) with an expected cost of $16,488.

Example 2. Review of Other Auditors' Work

Problem

Suppose that the Government Auditors' Office (GAO) is responsible for the audit of government corporations. However, because of a shortage of staff, GAO has contracted with commercial auditors to perform the audit of several government corporations. For these government corporations (i.e. "farmed-out" accounts), GAO intends to review the work of the commercial auditors contracted and issue the audit reports in the name of GAO instead of the commercial auditors. GAO can perform two types of review — extensive review (ER) or limited review (LR). The costs of ER and LR are $50,000 and $10,000, respectively. The ultimate concern of GAO is whether the financial statements of the "farmed-out" accounts are fair (F) or not fair (NF). Based on past experience, the probability of financial statements of government corporations being fair is about 0.85, i.e. Pr(F) = 0.85 and Pr(NF) = 0.15. Generally, the opinions of the commercial auditors are correct 95 per cent of the time in accepting financial statements that are fair and in not accepting financial statements that are not fair, i.e. Pr(A|F) = 0.95 and Pr(NA|NF) = 0.95.

GAO estimates that if the financial statements of a government corporation are not fair and it issues an unqualified audit report (UQ), then the expected cost of such an incorrect opinion is $100,000, i.e. E(C|NF,UQ) = 100,000. However, if the financial statements are fair and a qualified audit report (Q) is issued, then the estimated expected cost of such an incorrect opinion is $10,000, i.e. E(C|F,Q) = 10,000.

Results of the two types of review can signal whether the financial statements are fair or not. From past experience, in cases where the financial statements are fair and the commercial auditors accept the financial statements as fair, positive signals are received 90 and 80 per cent of the time respectively when ER and LR are performed, i.e. Pr(P|F,A,ER) = 0.90 and Pr(P|F,A,LR) = 0.80. The corresponding probabilities when the commercial auditors do not accept the financial statements as fair are Pr(P|F, NA,ER) = 0.85 and Pr(P|F,NA,LR) = 0.75. On the other hand, if the financial statements are not fair and commercial auditors do not accept the financial statements as fair, ER and LR give positive signals only 5 and 10 per cent of the time respectively, i.e. Pr(P|NF,NA,ER) = 0.05 and Pr(P|NF,NA,LF) = 0.10. The corresponding probabilities when the commercial auditors accept the financial statements as fair are Pr(P|NF,A,ER) = 0.20 and Pr(P|NF,A,LR) = 0.30.

Past records show that ER was performed 70 and 5 per cent of the time in cases where the financial statements were fair and (1) the commercial auditors accept the financial statements as fair, and (2) the commercial auditors do not accept the financial statements as fair respectively, i.e. Pr(ER|F,A) = 0.70 and Pr(ER|F,NA) = 0.05. In cases where the financial statements are not fair, the corresponding probabilities are Pr(ER|NF,A) = 0.50 and Pr(ER|NF,NA) = 0.25.

Given the information above, when should GAO auditors perform extensive review or limited review?

Solution

Given:

Pr(F)	= 0.85	Pr(NF)	= 0.15
Pr(A|F)	= 0.95	Pr(NA|NF)	= 0.95
Pr(ER|F,A)	= 0.70	Pr(ER|F,NA)	= 0.05
Pr(ER|NF,A)	= 0.50	Pr(ER|NF,NA)	= 0.25
Pr(P|F,A,ER)	= 0.90	Pr(P|F,A,LR)	= 0.80
Pr(P|F,NA,ER)	= 0.85	Pr(P|F,NA,LR)	= 0.75
Pr(P|NF,NA,ER)	= 0.05	Pr(P|NF,NA,LR)	= 0.10
Pr(P|NF,A,ER)	= 0.20	Pr(P|NF,A,LR)	= 0.30

Let Φ_i and Ω_j be defined as in Example 1. In addition, let Σ_k, for $k=1,2$, denote the events ER and LR, respectively; Γ_m, for $m=1,2$, denote the events A and NA, respectively. The given information corresponds to $\Pr(\Phi_i)$, $\Pr(\Gamma_m|\Phi_i)$, $\Pr(\Sigma_k|\Phi_i,\Gamma_m)$ and $\Pr(\Omega_j|\Phi_i,\Sigma_k,\Gamma_m)$. The conditional probabilities required for the decision tree can be derived by computing the quantities below sequentially.

Step 1.

$$\Pr(\Gamma_m) = \Pr(\Gamma_m|\Phi_1)\Pr(\Phi_1) + \Pr(\Gamma_m|\Phi_2)\Pr(\Phi_2).$$

Step 2.

$$\Pr(\Phi_i|\Gamma_m) = \frac{\Pr(\Gamma_m|\Phi_i)\Pr(\Phi_i)}{\Pr(\Gamma_m)}.$$

Step 3.

$$\Pr(\Sigma_k|\Gamma_m) = \Pr(\Sigma_k|\Phi_1,\Gamma_m)\Pr(\Phi_1|\Gamma_m) + \Pr(\Sigma_k|\Phi_2,\Gamma_m)\Pr(\Phi_2|\Gamma_m).$$

Step 4.

$$\Pr(\Phi_i|\Sigma_k,\Gamma_m) = \frac{\Pr(\Phi_i)\Pr(\Gamma_m|\Phi_i)\Pr(\Sigma_k|\Phi_i,\Gamma_m)}{\Pr(\Gamma_m)\Pr(\Sigma_k|\Gamma_m)}.$$

Step 5.

$$\Pr(\Omega_j|\Sigma_k,\Gamma_m) = \Pr(\Omega_j|\Phi_1,\Sigma_k,\Gamma_m)\Pr(\Phi_1|\Sigma_k,\Gamma_m) + \Pr(\Omega_j|\Phi_2,\Sigma_k,\Gamma_m)\Pr(\Phi_2|\Sigma_k,\Gamma_m).$$

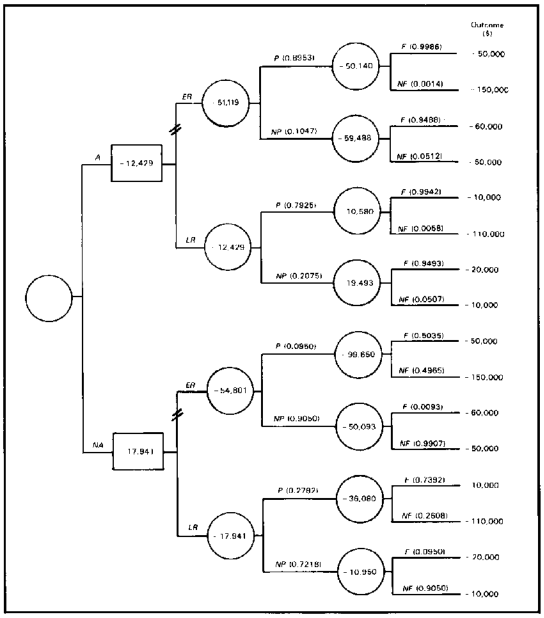
Step 6.

$$\Pr(\Phi_i|\Omega_j,\Sigma_k,\Gamma_m) = \frac{\Pr(\Phi_i)\Pr(\Gamma_m|\Phi_i)\Pr(\Sigma_k|\Phi_i,\Gamma_m)\Pr(\Omega_j|\Phi_i,\Sigma_k,\Gamma_m)}{\Pr(\Gamma_m)\Pr(\Sigma_k|\Gamma_m)\Pr(\Omega_j|\Sigma_k,\Gamma_m)}.$$

Using these relations and the given information, the following required conditional probabilities are derived:

Pr(P|ER,A)	= 0.8953	Pr(NP|ER,A)	= 0.1047
Pr(P|LR,A)	= 0.7925	Pr(NP|LR,A)	= 0.2075
Pr(P|ER,NA)	= 0.0950	Pr(NP|ER,NA)	= 0.9050
Pr(P|LR,NA)	= 0.2782	Pr(NP|LR,NA)	= 0.7218
Pr(F|P,ER,A)	= 0.9986	Pr(NF|P,ER,A)	= 0.0014
Pr(F|NP,ER,A)	= 0.9488	Pr(NF|NP,ER,A)	= 0.0512
Pr(F|P,LR,A)	= 0.9942	Pr(NF|P,LR,A)	= 0.0058
Pr(F|NP,LR,A)	= 0.9493	Pr(NF|NP,LR,A)	= 0.0507
Pr(F|P,ER,NA)	= 0.5035	Pr(NF|P,ER,NA)	= 0.4965
Pr(F|NP,ER,NA)	= 0.0093	Pr(NF|NP,ER,NA)	= 0.9907
Pr(F|P,LR,NA)	= 0.7392	Pr(NF|P,LR,NA)	= 0.2608
Pr(F|NP,LR,NA)	= 0.0950	Pr(NF|NP,LR,NA)	= 0.9050

Figure 6. *A Decision Tree to Evaluate Review of Other Auditors' Work*



Based on the computations above, the decision tree is presented in Figure 6. As can be seen, the conclusion that can be drawn from the decision tree is that whether the commercial auditors accept (A) or do not accept (NA) the financial statements as fair, the expected cost of performing limited review (LR) is lower than that of performing extensive review (ER). Therefore, performing limited review is the better decision.

Some Concluding Remarks

The auditing process involves sequential decision making and the ultimate cost of the whole process depends not only on the decisions taken at the various stages, but also on the outcomes following these decisions. In this context, a framework for the analysis and evaluation of the proper course of action to take at each stage of the decision-making process is clearly desirable. It is suggested here that a suitable framework is provided by the Bayesian decision-tree approach which is well documented in the statistical literature, but which hitherto has only found limited application in accounting.

Using this approach, it will usually be necessary to derive some conditional probabilities to be incorporated into the decision tree. While the two examples above show how these are derived in their contexts, the formulae used therein are usually not generally applicable. For a particular problem, the specific formulae used will depend on the information given. The following probability results, however, are generally helpful in deriving the required conditional probabilities.

- Suppose A_i ($i=1,2,\ldots,I$) and B_j ($j=1,2,\ldots,J$) are two different partitions of a sample space. Then:

$$\Pr(A_i|B_j) = \frac{\Pr(B_j|A_i)\Pr(A_i)}{\sum_{m=1}^{I}\Pr(B_j|A_m)\Pr(A_m)} = \frac{\Pr(A_i \cap B_j)}{\Pr(B_j)}$$

  for a given i and j.

- Suppose A_i ($i=1,2\ldots,I$) are events defined on a sample space. Then:

$$\Pr(A_1 \cap A_2 \cap A_3 \cap A_4 \cap \ldots) = \Pr(A_1)\Pr(A_2|A_1)\Pr(A_3|A_1 \cap A_2)\ldots.$$

- Suppose A and B are events, and C_k ($k=1,2,\ldots,K$) is a partition defined on a sample space. Then:

$$\Pr(A|B) = \sum_{k=1}^{K}\Pr(A|B \cap C_k)\Pr(C_k|B).$$

It can be seen from the two illustrative examples above that the amount of computation of the required probabilities increases substantially even with only a moderate increase in the complexity of the problem. In practice, this disadvantage loses much of its force if a computer program is written to perform the necessary computations. The program, once written, can be used to analyse and evaluate the problem when circumstances change, by altering the values of the appropriate parameters. It is hoped that this paper presents another tool which the practising auditor will find useful.

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